Ministry of Higher
Education and Scientific Research
University of Diyala
College of Science
Department of Mathematics

Improved Method Using Low Rank Matrix for Face Recognition

A Thesis Submitted to the Department of Mathematics / College of Science/ University of Diyala
In Partial Fulfillment of the Requirements for the Degree of Master in Mathematical Science

By

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2021AC **1443AH**

بِسْمِ اللَّهِ الرَّحْمَنِ الَّرحَيم

﴿ ٱللَّهُ ٱلَّذِى جَعَلَ لَكُمُ ٱلْأَرْضَ قَرَارًا وَٱلسَّمَآءَ

بِنَآءً وَصَوَّرَكُمْ فَأَحْسَنَ صُوَرَكُمْ﴾

سورة غافر

الاية (64)

Dedication

To Allah's Beloved One …

To the one who was sent as a clemency to the people …

Muhammad the Messenger of Allah's "Allah's prayers and peace be Him and His Household"

To my family …

My Father and Mother, the pulse of life that runs in my veins.

To my professors...

The candles that lighten my path of knowledge. I dedicate the fruit of my effort Honorably & Appreciatively to anyone who has had a good impact in the Completion of this modest work.

All our distinguished teachers paved the way for our science and knowledge.

Rusul Ismail Khalil

Acknowledgments

First of all, praise is to ***Allah*** *the lord of the whole creation, for all the blessing was the help in achieving this research to its end.*

I wish to express my thanks to my supervisor ***Dr. Rokan Khaji Mohammed*** *for supervising this research and for the generosity, patience, and continuous guidance throughout the work. It has been my good fortune to have advice and guidance from him. My thanks are to the academic and administrative staff at the Department of the Mathematics sciences\University of Diyala for their hospitality and generosity.*

I would like to express my gratitude to ***my father, my mother, my brothers, and my sisters. Many thanks For is support Special thanks are to Dr.Dheyaa Alangood.*** *All my friends and to all lovely people for their great support and continuous encouragement, directly or indirectly to complete this work.*

(Supervisor’s Certification)

We certify that this research entitled “Improved Method Using Low Rank Matrix for Face Recognition” was prepared by **Rusul Ismail Khalil** under our supervision at the University of Diyala Faculty of Science Department of Mathematics Science, as partial fulfillment of the requirement needed to award the degree of Master of Science in Mathematics Science.

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I certify that this research entitled “Improved Method Using Low Rank Matrix for Face Recognition” was prepared by Rusul Ismail Khalil and was reviewed linguistically. Its language was amended to meet the style of the English language.

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List of Abbreviations and Symbols

Abbreviations	Meaning
RPCA	Robust Principal Component Analysis
PCA	Principal Component Analysis
SRC	Sparse Representation-Based Classification
CRC	Collaborative Representation-Based Classification
LR	Low Rank
LR-CRC	Low Rank Collaborative Representation
LR-SRC	Low Rank Sparse Representation
SVD	Singular Value Decomposition
TILT	Transform Invariant Low-Rank Textures
MTJSRC	Multiple Co-Sparse Representation is Proposed Tasks
BDDL	Binary Discriminant Dictionary Learning
PCP	Principal Component Pursuit
ALM	Augmented Lagrange Multipliers
ORL	Olivetti Research Laboratory
NN	Nearest Neighbor
ELM	Extreme Learning Machine
ELMSRC	Extreme Learning Machine with Sparse Representation based Classification
ASM	Active Shape Model
AAM	Active Appearance Model
D	Big Matrix
X	Low-Rank Component.
E	The Noise or Error in Measurements.
$\|.\|_F$	Frobenious Norm, $\| Y \|_F = \sqrt{\sum_{ij} Y_{ij}^2}$
X^T	Transpose of Matrix or Vector
u_i	The Left Singular Vectors.
v_i	The Right Singular Vectors.
σ_i	Singular Values of D

Abstract

Face recognition generally refers to a class of methods that solve problems by representing variables of interest as low-rank matrices. It has achieved great success in various fields including computer vision, luminance variations, severe expression variations, random pixel corruption, contiguous occlusion, and bioinformatics.

Recently, much progress has been made in theories, algorithms and applications of low-rank for Face recognition, such as low-rank matrix for face recognition applied to sparse representation, collaborative representation. These advances have brought more and more attentions to this topic. In This thesis, proposes we review the recent advance of low-rank for face recognition, the state of-the-art algorithms, and related applications in image analysis. We first give an overview to the concept of low-rank for face recognition and challenging problems in this area. Then, we summarize the models and algorithms for low-rank matrix in face recognition and illustrate their advantages and limitations with numerical experiments. Next, we introduce a few applications of low-rank for face recognition in the context of image analysis

Chapter One

Literature Review

CHAPTER ONE

Literature Review

1.1 Introduction

Identity management is a critical function in a variety of applications in the field of modern information management technology, making inseparable components from it. One such application is the protection and confidentiality of information, which includes restricting access to data and information at nuclear power plants, airports, and banks, as well as managing logical access to resources related to then in addition to, organizing border crossings to neighboring countries, conducting financial transactions remotely, or social distribution. Social welfare services, web-based services, and decentralized customer service centers (e.g online banking and credit card services). All the mention applications and many more need highly trusted identity management systems to reduce the risk and mistrust regarding information security, confidentiality, and identity theft.

The process of determining the relationship between individuals and their identities is the primary task of identity management. A person can be identified by identity, to validate their claim. Personal identification is defined as the process of identifying people using one of the three basic methods listed below: First, they have exclusive confidential information (for example, a password, a PIN, or an encryption key), this approach is what the person knows. The second suggests, the exclusive possession of the external token should be through the person who owns the object (e.g., ID card, driver's license, passport, physical key, or personal device such as a mobile phone), and this approach is what the person owns externally. The third approach, who is a core person, can be identified by their inherent physical or behavioral traits and is known as biometric identification or field scanning.

Biometric recognition examines a person's unique physical or behavioral characteristics in a fully automated or semi-automated way to identify the person. There are a variety of reasons, knowledge-based and token-based mechanisms, such as surrogate representations of identity such as passwords or PIN or ID cards, have not been adequate to manage a trusted identity, which cannot be remembered/lost/easily stolen or similar, along with a fraudulent presentation or redundant.

Documents can be also allow the identification of individuals to be concealed. Using identification, or simply biometrics, it is possible to confirm or establish the identity of an individual. Since vital characteristics are inherent in a person, these traits cannot be manipulated, shared, forgotten, and cannot be manipulated. That is users of a biometric system who provide their biometric ID to a system identify it. Figure (1.1) shows some basic methods for identifying people.

Figure (1.1) Some basic approaches for person recognition.[1]

The biometric system extracts features set from the acquired biometrics data, andcompares it with the template sets in the data and for many decades it has been and still is an area of extensive research. The biometric traits includes, fingerprint, palmprint,face, iris, retina, ear, voice, signature, gait , hand geometry and the DNA information of an individual to determine or verify his identity etc,All this typies describe it and put in the end them[1-12]. as shown in figure (1.2).

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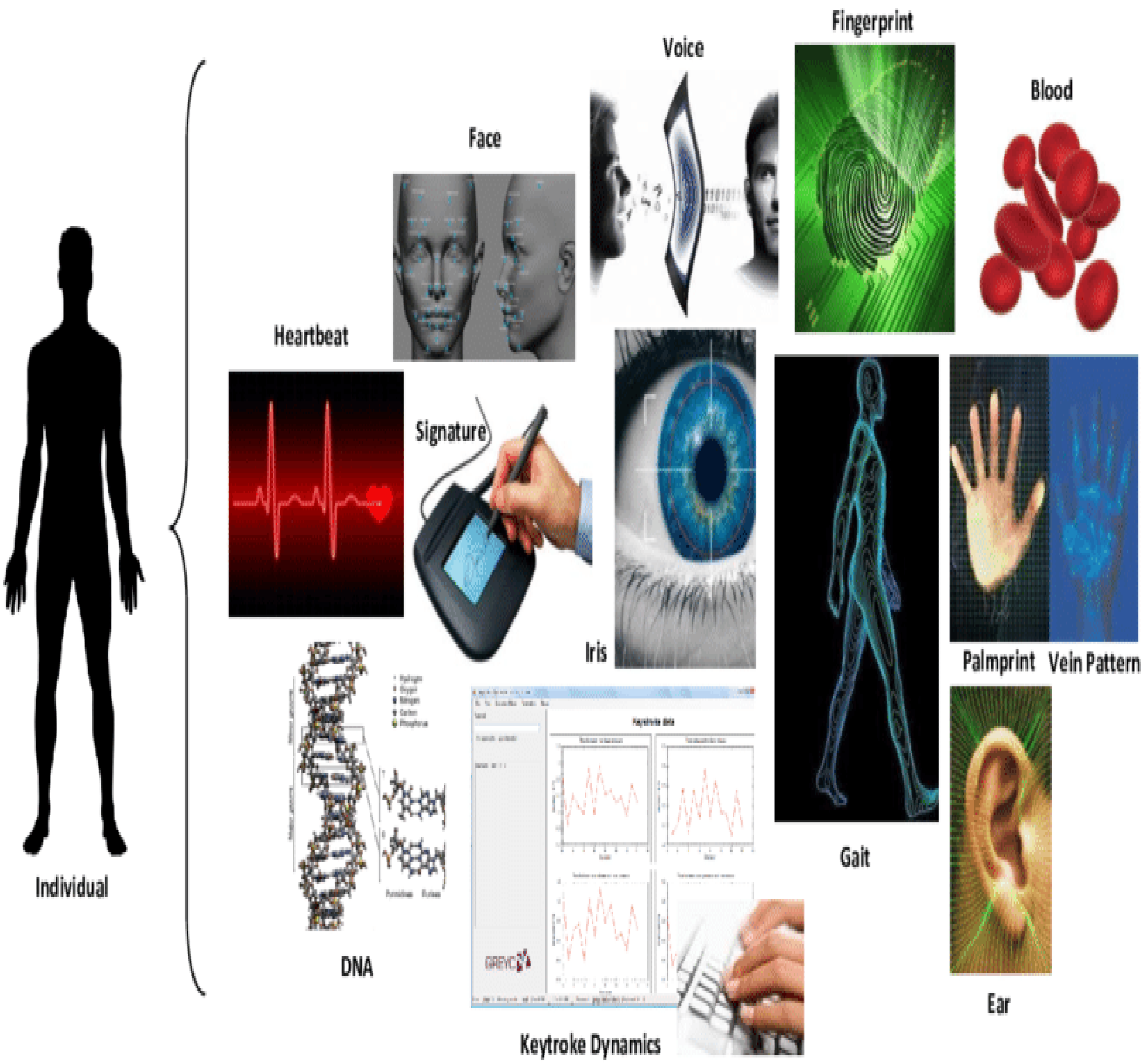

Figure (1.2) Some examples of body traits that have been used for biometric recognition[12].

Cognitive biometrics refers to face recognition, as it has many advantages over other biometric methods, some of which we explain here: Most of these technologies requires the user to work with some voluntary action, i.e. placing a hand on a fingerprint device to take fingerprints or detect The shape of the hand damage to the skin tissue of the hands and fingers (i.e. bruised or cracked) this may led to, to identify the iris or retina, the person must stand in a fixed position in front of the camera .

As for those who suffer from eye diseases such as damage to the retina or iris, this might prevents utilizing in the techniques that depend on it, and resorting to other techniques. In the identification of the iris and retina, these techniques need advanced devices with high technology and very high prices and are very sensitive to any movement of the body. High-noise sound in public places and fluctuations of sound can be recognized on a mobile phone or recorded on tape.

Taking a swipe for signature through an automated scan that can be modified or forged. In the case of capturing biological characteristics, through heavy use the device can become a transmitter of germs and impurities from one user to another this require continuous sanitizing as experienced during Covid-19 pandemic, while faces cannot be recognized properly without any explicit action or user intervention since the face images. Can be obtained easily with two fixed cameras and it is cheap. Facial recognition is one of the most important biometric technologies and has the advantages of high accuracy and low intrusion, and it is important in many real-world applications (for example, access control, Mobil phones, information protection and confidentiality, surveillance, and entertainment, smart cards and human-computer interaction (HCI) communities) [13].

In addition to its usefulness in person recognition, the facial images are a suitable also for revealing other attributes like biographic information (e.g. age, ethnicity, and gender) and emotional state of a person (e.g., happiness or anger).

Face recognition can be defined as the process of establishing the identity of a person based on facial characteristics [13].

In another simple phrase, it's a comparing between two of face images and determining if they are of the same person. The recognition task is a challenging with the face appearance of a person due to many variations between the images of the same face (e.g., age, pose, illumination, and facial expressions as well as exhibit changes in appearance due to make-up, facial hair, or accessories) as shown in figure (1.3).

Figure (1.3) Some examples of face feature [13].

These challenges are accompanied by progress in the field of automatic face recognition; however, facial recognition is still having some problem and the solution to this problem has not been completed correctly until this moment. There is a wide field for researchers to identify the characteristics and features that these images possess and to identify them more through experiments and their comparison with the previous studies.

1.2 History

1. **Z.Ningbo , et al. [72], 2013,** introduced a method to improve the classification accuracy of face recognition, a sparse representation method based on kernel and virtual samples is proposed. The proposed method has the following basic idea: first, it extends the training samples by copying the left side of the original training samples to the right side to form virtual training samples. Then the virtual training samples and the original training samples make up a new training set and we use a kernel-induced distance to determine M nearest neighbors of the test sample from the new training set. Second, it expresses the test sample as a linear combination of the selected M nearest training samples and finally exploits the determined linear combination to perform classification of the test sample
2. **R.Abiantun ,et al. [73], 2014**.Showed that focus on one-to-one matching scenarios where a query face image of a random pose is matched against a set of gallery images. We propose a method that relies on two fundamental components: (a) A 3D modeling step to geometrically correct the viewpoint of the face. For this purpose, we extend a recent technique for efficient synthesis of 3D face models called 3D Generic Elastic Model. (b) A sparse feature extraction step using subspace modeling and $\ell 1$-minimization to induce pose-tolerance in coefficient space. This in return enables the synthesis of an equivalent frontal-looking face, which can be used towards recognition. We show significant performance improvements in verification

rates compared to commercial matchers, and also demonstrate the resilience of the proposed method with respect to degrading input quality. We find that the proposed technique is able to match non-frontal images to other non-frontal images of varying angles.

3. R.Jafri, et al. [15], 2015. proposed Sparse representation based classification (SRC) produces interesting results for robust face recognition by coding a query sample as a sparse linear combination of all training samples and then classifying it by evaluating which class leads to the minimal coding residual.

4. R. Vidal, et al. [9], 2016. Assume we have a data matrix consisting of a low-rank component and a sparse component superimposed. Is it possible to restore each component separately? We show that by solving a relatively straightforward convex program called Principal Component Pursuit, it is possible to recover both the low-rank and sparse components perfectly under some reasonable assumptions; among all viable decompositions, just minimize a weighted mixture of the nuclear and l -norms

5. N.Oliver,et al. [45]2017.Introduced a method the kernel minimum square error classification (KMSEC) algorithm has been widely used in classification problems. It shows a good performance on image data besides the following drawbacks: not sparse in the solutions and sensitive to noises. The latter drawback will result in a decrease in the recognition performance. We propose an improved kernel minimum square error classification algorithm (IKMSEC) by using the $L2,l$-norm -norm regularization, which can obtain a sparse representation of nonlinear features to guarantee an efficient classification performance. The comprehensive experiments show the promising results in face recognition and image classification.

1.3 Motivations

The researcher was to get each person (the identity of the person) quickly and simply through modern and advanced technologies, but in the past, this goal was difficult. The easiest and simplest feature in people that can be recognized, understood, and cannot be forgotten is the face where natural biometric techniques are can smoothly recognize, In the past researchers used optical technology to identify people, The process of face recognition became important all over the world and the need for automatic face recognition raised.

Recently, studies and some applications have turned to face recognition. Some algorithms are optimized to solve the problem of the effect of a single factor or combination of appearance difference factors for face recognition. For example, some high-end features that work with sensitive lighting conditions have been optimized and tested on a set of data that contains only different lighting. Finally, a database of faces was obtained under controlled settings that contain differences in facial appearance caused by one or more of a combination of factors. The value of these studies is to solve the problem of performing facial recognition and to know the power of the algorithms to collect these databases. It is necessary and important to test every single face algorithm on these standard face databases and even

- Through one or two sources, several differences are created in the appearance of the face, so that the image of the face can be directed, measured and photographed differently.
- There are several differences in the appearance of the face that are separate, for example the positions of the head are directed at a certain angle or from different directions.
- When a person wants and realizes that a picture has been taken of him, he must remain in front of the camera. It is collected collaboratively.

The universal human trait is in the face. The process of face recognition is not only important but because of the potential of many potential applications in the fields of research, and solving classification problems is important and can solve this matter because of the ability such as object recognition.

1.4 Challenges of Face Recognition

Face recognition has attracted a lot of attention in more than three decades and many studies have been carried out for it. But there are still many challenges facing. It the most difficult aspects of face recognition are summarized as follows:

1. There are an enormous number of humans: when identifying similar faces, Different persons may have similar appearance that sometimes it is impossible for a human to identify them, moreover, if they are genetically related (identical twin, father and son, etc.). Such the inter classes similarities further increase the difficulty of face recognition as shown in figures (1.5), (1.6).

Figure 1.4 Some examples identical twin image [15].

Figure 1.5 Some examples similarly son and father [15].

2. Illumination: Illumination means light variations. Illumination changes can vary the overall magnitude of light intensity reflected from an object, as well as the pattern of shading and shadows visible in an image. Indeed, varying the illumination can result in larger image differences than varying either the identity or the viewpoint of a face. The same individual imaged with the same camera and seen with nearly the same facial expression and pose may appear dramatically different with changes in the lighting conditions.

The problem of face recognition over changes in illumination is widely recognized to be difficult for humans and for algorithms. The difficulties posed by variable illumination conditions, therefore, remain a significant challenge for automatic face recognition systems. It is found that the difference between two images of the same person taken under varying illumination is greater than the difference between the images of two different persons under same illumination. The variation in illumination changes the appearance of the face drastically as shown in figure (1.7).

Figure (1.6) Some examples Variations in illumination[15].

3. Effect of the external factors on face appearance: When in a Face Recognition System the whole face is not available as input image or image sequence, then it is termed as Occlusion. It is one of the important challenges of the face recognition as shown in the figure (1.8).

This is due to presence of various occluding objects such as glasses, beard, moustache etc. on the face and when an image is captured from a surveillance camera; the face lacks some parts. In real world applications also, it is very common situation to acquire persons talking on the phone or wearing glasses, scarves, hats, etc. or for some reasons having their face covered with hands. Such a problem can severely affect the classification process of the recognition system.

Figure (1.7) Some examples Face occlusion and disguise [16].

4. Effect of the factors of pose, expression and aging on face appearance: The human face is not a unique, rigid object. Everything changes with time, so with the increasing age the appearance of a person also changes which affect the face recognition system, a face appears in many different shapes viewed from different angles in an image. The changes in facial features as a result of aging factors (wrinkles, speckles and sagged cheeks, eyes, or mouth) as shown in figure (1.9).

Figure (1.8) Some examples many different shapes from different angles in an image [17].

1.5 Face Recognition Techniques

Face recognition system, as one of the fundamental biometric technologies, became more important owing to rapid progress in technologies such as digital cameras, the Internet and mobile devices, and increased demands on security. The beginnings of face recognition technology were in 1960s [16], where developed the first automated face recognition system required the administrator to locate features on the photographs before calculating distances and ratios to a common reference point, which were then compared to reference data, and then the techniques expansion and developed rolled until it reached that it is today.

There are two types of techniques in the elementary stage applied on Face recognition with views of the front. The first is the earliest techniques to face recognition [17], which are based on the computation of a set of geometrical features (such as forehead, eyebrows, cheeks, chin, eyes, ears, nose, and mouth) by detecting the significant facial features and the distances among them as well as other geometric characteristic that are combined in a feature vector that is used to represent the face.

The second is represented as the most popular technique to recognize and detect faces [18] that is based on template matching and it uses a feature vector that represents the entire face template rather than the most significant facial features that are unlike the geometric technique.

In recent years, the face recognition has received a growing interest through holding face recognition conferences and many systematic empirical evaluations of face recognition techniques, and this was reflected by significant developments which witnessed face recognition techniques. The main face recognition techniques can be .Broadly divided into four categories based on the face data acquisition methodology as outlined below:

1. Appearance-based approaches generate a compact representation of the entire face region in the acquired image by mapping the high-dimensional face image into a lower dimensional sub-space which is defined by a set of representative basis vectors, and these vectors are learned using a training set of images. Principal component analysis (PCA) and linear discriminate analysis (LDA) are the appearance-based approaches represented by Eigenfaces and Fisherfaces [19, 20] algorithms respectively, such as based methods, and are significantly advanced face recognition techniques. Although these linear, appearance-based methods (e.g., Eigenfaces [19] and Fisherfaces [20]) have been widely used, they are not accurate enough to describe subtleties of original manifolds in the original image space. This is due to their failure to reveal the essential data structures nonlinearly embedded in high dimensional space: To overcome this problem such linear methods can be extended using nonlinear kernel techniques (kernel PCA [21] and kernel LDA [22]) to deal with nonlinearity in face recognition [23].

2. Model-based techniques are an attempt to build 2D or 3D face models that facilitate matching of face images in the presence of pose, expression, illumination and age variations. Recently, model-based technologies have become available that includes: The statistical approach which is called face alignment models is adopted to learn the way in which the face shape and texture vary across a large and representative training set of face images from these models is Active Shape Model (ASM) [24] and Active Appearance Model (AAM) [25]; The morphable face model which aims to build a synthesis framework, which is able to generate all possible face images.

3. Texture-based approaches, which are used to find robust local features, that include face recognition based on gradient information, which is an insensitive and robust to different illuminations, and face recognition based on local statistical

features such as holistic feature based (Eigenface and Fisherface) [19, 20], these schemes achieve very promising face recognition results.

4. Representation based approaches, which aim at extracting features based on linear combination of training images or their features. The recognition is achieved by testing which class of the sufficient expressive training set could result in the minimal distance between the testing face image sample and it. In this approach, the labels of training samples to represent the testing image by collaboratively across different classes or belong to the same class.

1.6 Objectives and Scope

The study in this thesis was focuses on developing novel and robust recognition algorithms for face recognition low rank matrix, that works in general frame which is sparse representation. Recent developments and extensions in the theory of sparse coding has found numerous applications in various fields of signal processing, The development of computer vision and pattern recognition has presented many challenges in terms of face recognition. This thesis addresses the challenging issues of face recognition under different conditions as the variation in illumination, occlusion, pose, expression, etc., this thesis addresses these issues in a practical manner. There have been a number of methods presented in an attempt to improve the extraction of features from face recognition in this paper, the proposed method is proposed to enable the down sampling of images to be utilized for the feature extraction stage.

As mentioned earlier from in this section, sparse representation-based classification (SRC) was the general frame for this work. We successfully improved three approaches which essentially are an extension of sparse representation-based classification (SRC) for other challenging problems of view-based face recognition systems. The scope of this thesis includes the following:

• Collaborative representation-based classification for face recognition is based on bilateral filtering and has been introduced various computational random collections (CRCs) that can be used for face recognition. Then they are integrated into one algorithm to provide a suitable model for face recognition.

•Robust principal component analysis dictionary is utilized for sparse representation-based face recognition, the original image samples have a lot of redundant information and noise. This issue can be overcome by carrying out robust principal component analysis at the same time as training samples and at the same as the training samples are huge and in order the computation of sparse representation will not be time consuming.

• Finally, a new simple and perfect algorithm has been proposed for face recognition, where this algorithm integrates the low-rank matrix which is recovered by using robust principal component analysis (RPCA), and this thesis aims to develop novel representations-based classification models for face recognition that are more robust and provide better information for face identification. The detailed objectives of this thesis are as follows:

• To develop a novel face recognition system called collaborative representation (CRC), we combine it with low rank collaborative representation (LR-CRC).

• To develop efficient collaborative representation-based classification model by integrating it with bilateral filter and to customize this model within the face recognition framework.

Analysis for the challenging issues:

(1) Illumination variations.

(2) Random pixel corruption.

1.7 Thesis Organization

The thesis is organized in the following way:

Chapter 2: It will present a description of efficient low rank matrix for face recognitions and algorithms that can be used to learn sparse representations from the input data, which can also help the classification tasks.

Chapter 3: It introduces the new extension of the SRC algorithm for the face recognition problem, which is combination of RPCA for Sparse Representation. experiments have been conducted and results are compared with the state of art methods.

Chapter 4: It introduces Collaborative Representation for Face Recognition based (CRC) algorithm which is a new method for the problem of face recognition. It is important to note that it was successfully applied to the different face images database.

Chapter 5: Conclusions and Future work provides concluding remarks and suggestions for future work.

CHAPTER TWO

THEORETICAL BACKGROUND

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THEORETICAL BACKGROUND

2.1 Introduction

To make a face recognition system, one typically concentrates on the learning of classification models and the extraction of facial specifications.to evaluate verification performance, unseen test data from the same subjects will be used. Occlusion and disguise might be presented in the test image data to assess the robustness of the designed face recognition algorithm. It is important to notice if the test data is corrupted, the training data is often assumed to be taken under a well-controlled sitting (i.e., under good pose, illumination, etc. variations without disguise or occlusion).

The need to discard corrupted training images is one of the results of applying existing face recognition methods for practical scenarios, this would make us encounter over-fitting and small sample size problems. More than that, some valuable information for recognition of face recognition might be given by the disregard of corrupted training face images, usually, to reduce the dimension of the face images we would use some existing techniques such as Eigenfaces [19], Fisherfaces [20], or Laplacianfaces [26]. It is expected for the derived subspace to achieve improved recognition performance. Anyway, these techniques are not sparse/extreme or robust noise such as disguise and occlusion [27]. Latest research on robust Principal Component Analysis (PCA) suggested to reduce the problems showed in [28, 29, 30]. A promising result would be shown by solving low-matrix recovery in polynomial-time [28].

The remainder of this chapter is organized as follows: In Section 2.2, we review briefly some related works on low rank matrix Model; in Section 2.3 Applications of Low Rank Matrix Model; in Section 3.4 we give an overview of Procedure of Face Recognition

2.2 Low Rank Matrix Model.

Generally, Low Rank modeling refers to a class of methods used to solve problems by representing the interested variables as low rank matrices. Low-Rank modeling used in many fields like data mining, signal processing, bioinformatics, and Computer vision. Mush progress has been made recently in theories, algorithms, and applications of low-rank modeling, like matrix completion applied to collaborative filtering.

Due to these advances research have brought more attentions to this topic. Often, the data to be analyzed in many fields of research have high dimensionality, which make big problems to analyze the data. Some examples are customers' records in recommender systems, documents in natural language processing, images in computer vision, and genomics data in bioinformatics. The data with high dimensionality often lie in a subspace of low dimensionality.

In a mathematical way, if we make a representation of each data point to a vector $d_i \in R^m$ for $i = 1, \dots, m$,and indicating all of the dataset to a big matrix $D = [d_1, \dots, d_n]$ the following low-rank assumption is resulted from the low-dimensionality assumption:$rank(D) \ll min(m, n)$. In computer vision, Lambertian reflectance is a typical example, where $d_1, \dots, d_n$ correlated with a set of images of a convex Lambertian surface under lighting conditions [31]. We can also include another example from signal processing, where $d_i$is stand for the vectors of signal intensities received by an antenna array at time point i [32]. Because of the noise, the raw data can hardly be perfectly low rank in the real world, so that the following model id more realistic:

$$D = X + E \tag{2.1}$$

Where:

X: Low-rank component.

E: The noise or error in measurements.

Because of that, centric task in many problems became recovering the low-rank structure from noisy data. A traditional way to finding the low-rank approximation is made by solving the following optimization problem:

$$\min_{x} \| D - X \|_F^2 \quad , \quad s.t \quad rank(X) \leq r \tag{2.2}$$

where: $\| Y \|_F = \sqrt{\sum_{ij} Y_{ij}^2}$ Which it stands the Frobenious norm of matrix. When we solve such a minimization problem, it can be interpreted as seeking the optimal rank-r estimate of D in a least square way.as claimed by the matrix approximation theorem [33], the solution relation (2.2) is given in analytical way by the singular value decomposition (SVD)

$$X^* = \sum_{i=1}^{r} \sigma_i \, u_i \, v_i^T \tag{2.3}$$

where:

u_i: The left singular vectors.

v_i: The right singular vectors.

σ_i: Singular values of D.

For $i = 1, \ldots, r$

The vectors $u_1, \ldots, u_r$ from an orthogonal basis to regard an r-dimensional subspace that can best firmed the data. In statistics, this procedure correlated with Principal Component Analysis (PCA) [34]. Because of the provable optimality under certain assumptions and the analytical solution in computation, the PCA is one of the most popular tools for data analysis. This analysis cannot solve some problems in real applications. We can mention here two common examples:

- **Recovery from Gross Errors:** In some cases, recovering a low dimensional subspace from corrupted data is a basic task. For example, the true appearance might be occluded by a glasses or shadows of a person in his face images. The classical PCA supposes identically and independently distributed (I.I.D) Gaussian noise and adopts the sum of squared differences as the loss function as shown in relation (2.2).

  The least-squares fitting is so sensitive to outliers so that it is so easy for these gross errors to corrupt the classical PCA. As an example, because of the glasses or shadows in the input images of the reconstructed face images an artifact could be included. The researchers' attention focused on recovering a subspace or low rank robustly in the presence of outliers. ***Robust Principal Component Analysis (RPCA)*** is the named often called for this problem[29].

- **Recovery from a Few Entries:** For many applications, it is very important to recover a matrix from only a small number of observed entries. A very common example is that, when building recommender systems, we wish to make predictions to customers' preferences based on the information collected so far. We can show here a famous instance like the Netflix problem. Where the data is a big matrix D with each entry can be given as follows: $D_{ij}\{1, \ldots ,5\}$, where i the rating of customer for movie j.In the dataset there are around 480K customers and 18K movie.

  In average, each customer only rated about 200 movies, which means only 1.2% entries have values. Predicting the ratings that have not been made yet based on the current observations is the problem.one of the popular solutions is to suppose that the rating matrix should be low rank. This assumption made in the truth that subgroups of customers are expected to have the same taste and their ratings to the movies will be with high agreement. In consequence, the number of subgroups formed by the customers will bound the rank of the rating matrix. This problem is noted as ***Matrix Completion (MC)*** [35].

2.3 Applications of Low Rank Matrix Model

Low-rank matrices could be a model of many important objects in image analysis like dynamic textures changing periodically [36], the images of a convex Lambertian surface under various illuminations [31], active contours with similar shapes [37] , and multiple feature tracks on a rigid moving object [38].

The common patterns underlying the data in intuitive way are the low-dimensional subspace models. So that, it is important for many applications to recover the low-rank structure like face recognition, background subtraction, and segmentation. We will show in the following discussions some applications based on the low-rank matrix model in the field of image analysis.

2.3.1. Face Recognition

For many decades since the work made by Sirovich and Kirby [39], low dimensionality concept had been used in face recognition. Applying PCA give the ability for each face image to be characterized by a low-dimensional vector and to construct a face space and [39, 40]. Subsequently, the "*eigenface"* method for face recognition introduced [41]. One of the earliest examples to use low-rank modeling for face recognition by using eigenface method is as follows:

1- Computing the first N eigenvectors of the matrix composed of a set of training images which gives the ability to generate N eigenfaces.
2- Projecting the image onto the space spanned by the N eigenfaces to calculate the weight vector of the input image.
3- Checking if the input image is a face or not, where if it is a face, determine the weight vector and the person face according to error.

Usually, different artifacts like occlusions, specularities and shadows would corrupt the face images in real datasets, such artifacts cannot be handled by classical PCA. So that, many attitudes made to process face images based on RPCA [42- 44]. One of the strategies made to boost the performance of recognition algorithms and improve the characterization of faces is to remove the local defects in face images as the sparse component, while the correct description of the person's face could be obtained from low-rank component as shown in figure (2.1).

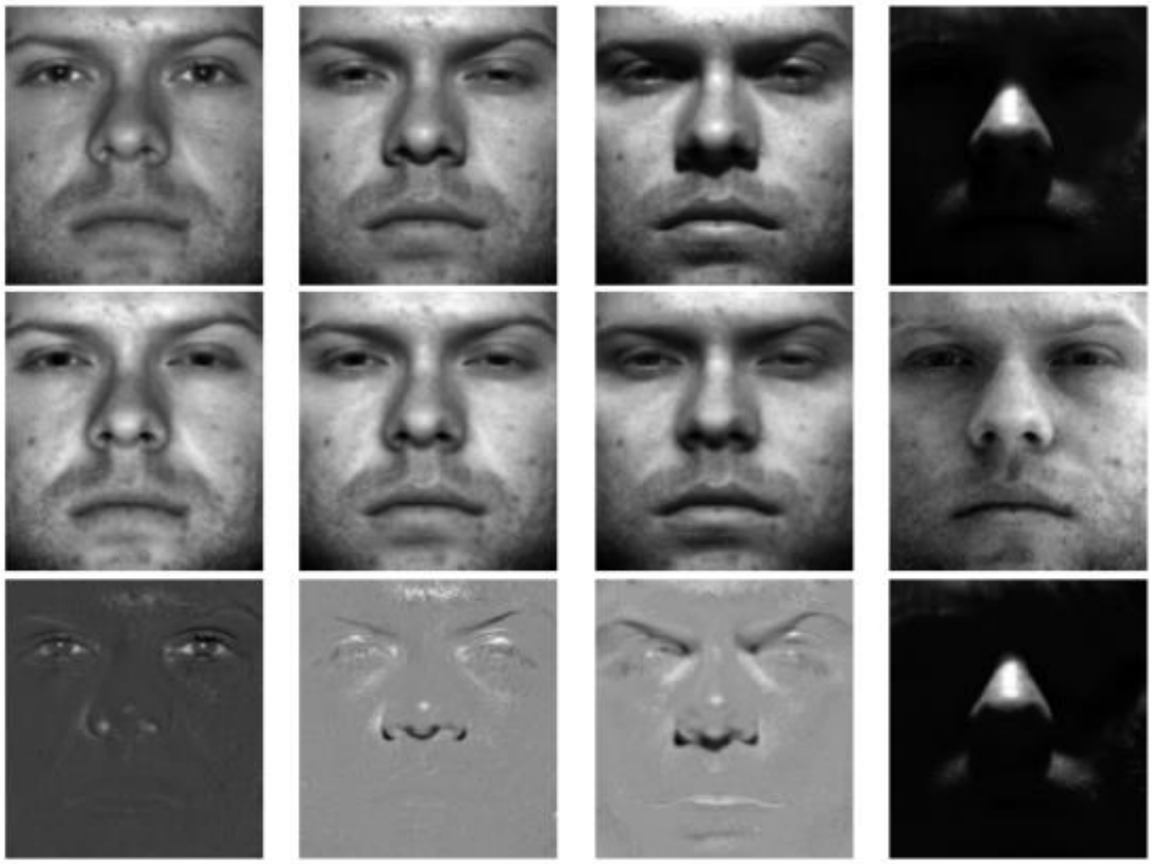


Figure2.1 Some example RPCA to remove shadows and specularities in face images [43].

2.3.2. Background Subtraction

Detecting the objects that stand out from the background in a video and modeling the background are included by the Background subtraction. Like eigenfaces, PCA has been applied to model the background since the work “eigenbackground subtraction" [45]. Unchanging the underlaid background images of a video captured by a static camera except for illumination variation is the essential idea of eigenbackground subtraction. it is required to Hence, it can be naturally model the matrix composed of vectorized background images as a low-ran matrix. Anyway, to generate a clean background model in traditional methods, make a set of training

images without foreground objects. A desired method to predict a background model at the presence of foreground objects is the RPCA method [29].
The PCP algorithm can identify the foreground objects in the sparse component and recover the background images in low-rank component as shown in [28]. Figure 2.2 gives a good clarification.

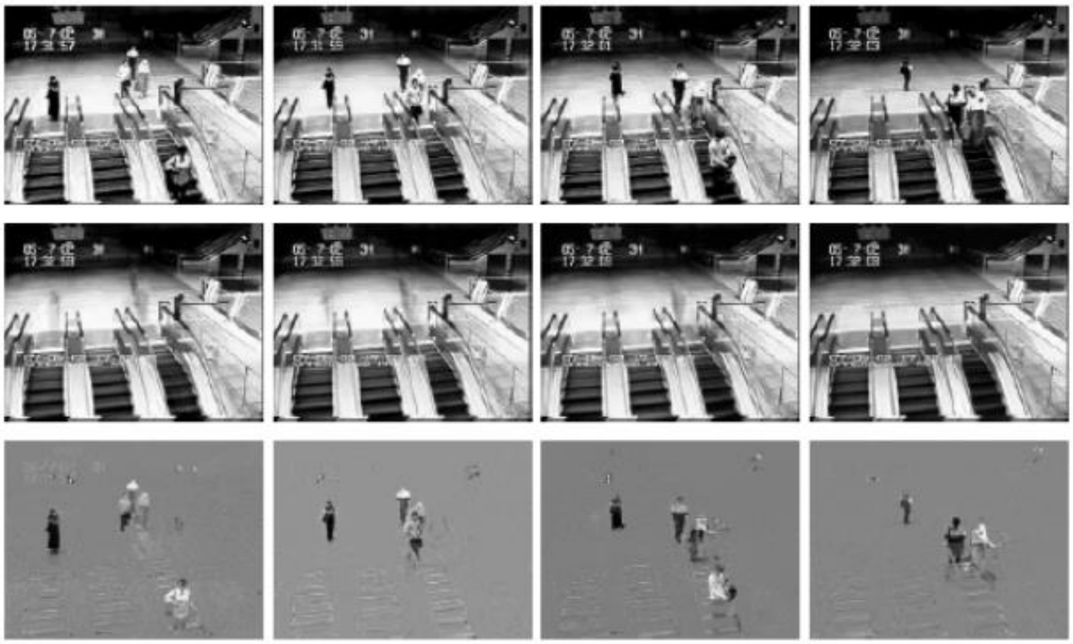


Figure 2.2 Some example RPCA for background subtraction [47].

By noticing the background which includes three moving escalators, these escalators are clearly reconstructed in the low-rank component. This shows the appealing capability of low-rank modeling for background subtraction. The spatially contiguous property of foreground pixels can be modeled and integrated into RPCA by using Markov Random Fields or other smoothing techniques to achieve better accuracy for object detection [46-48]. To segment the point trajectories in a video into two groups, which correspond to background and foreground, RPCA framework could be used [49]. The segmentation is because the camera motion results the background motion which should lie in a low dimensional subspace.

2.3.3. Image Segmentation

To increase the robustness of deformable models for image segmentation, the active shape model [50] was suggested. This model builds a statistical shape space from a huge set of given shapes and constrains the candidate shape in the shape space. A candidate shape could be represented by the next equation:

$$C(w) = \bar{C} + \Phi w \tag{2.4}$$

Where:

$\bar{C}$: The mean shape.

Φ: Matrix consisting of vectors describing shape variations in the training data.

w: Vector of coefficients to represent the candidate shape in the shape space.

w is determined by fitting the parametric curve in relation (2.4) to the features in the image. According to the fact that the number of columns of Φ is often small, the candidate shape is confined in a low-dimensional space. Hence, the active shape model admits a low-rank assumption on the population of shapes intrinsically. Anyway, C and Φ are derived by applying PCA to the set of training shapes. The active shape model was later extended to the active appearance model to make use of both shape and appearance information [50].

More exemplar methods building upon the active shape model for image segmentation include [51-54]. Imposing a group similarity constraint on multiple shapes by nuclear norm minimization is an alternative approach to make use of the low-rank assumption for image segmentation which does not require training shapes [55].

2.3.4 Image Alignment and Rectification

The problem of transforming different images into the same coordinate system called image alignment. The assumption of a batch of aligned images should form a low-rank matrix is a proposed solution for the problem by rank minimization based [56]. By solving equation (2.5), the parameters of transformation ij were estimated:

$$\min_{T,X,E} \| X \|_* + \lambda \| E \|_1 \quad , \quad s.t. \;\; D_{ij} = X + E \tag{2.5}$$

Where each column of D corresponds to an image to be aligned D_{ij} the images after transformation. E is the sparse component, which models local differences among images.

Similarly, the model in relation (2.5) is used to generate transform invariant low-rank textures (TILT). The difference between [56, 57] is that D in TILT represents a single image instead of an image sequence. The assumption of TILT is that the rectified images of textures such as bar codes, characters, and urban scenes are usually symmetric patterns and consequently form low-rank matrices. The reconstructed low-rank texture can be further used in many applications such as character recognition, camera calibration and 3D reconstruction, etc.

2.3.5 More Applications

There are many more applications of low-rank model include image restoration and denoising, object tracking, image restoration and denoising, medical image reconstruction, correspondence estimation, and model fusion [56-62].

2.4 Procedure of Face Recognition

In general, the face recognition system consists of the following steps:

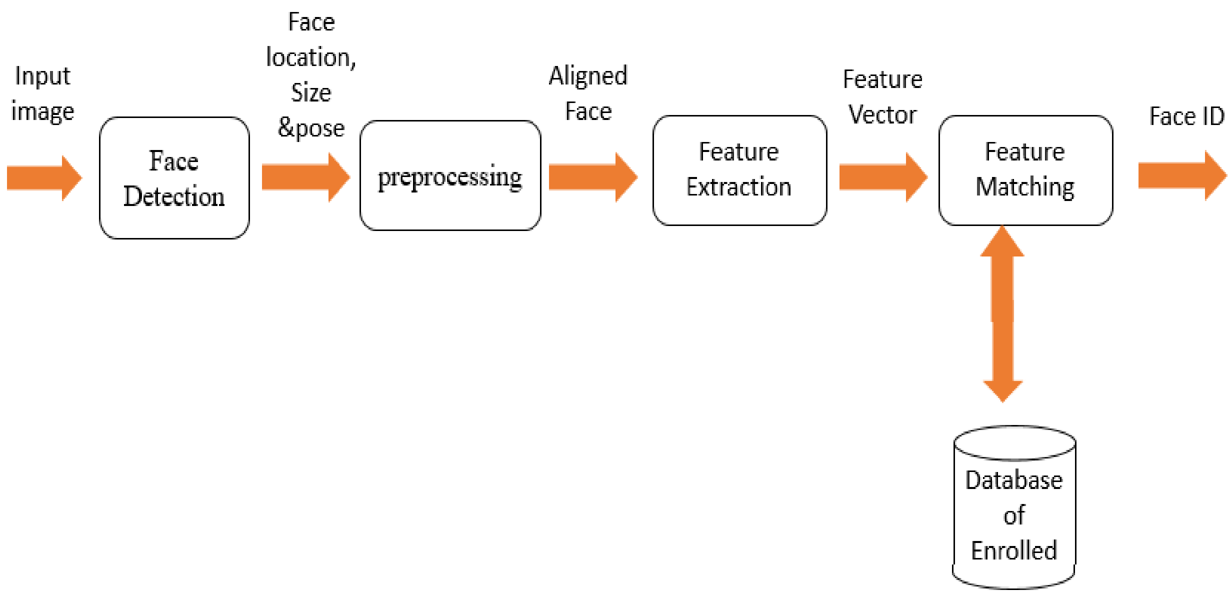
1. Face detection: It is the first step in the recognition system, the camera detects and locates the image of a face, either alone or in a crowd. The image may show the person looking straight ahead or in profile.

2. Preprocessing: It reduces noise, and the face area is cropped, aligned, or normalized from face images, to facilitate feature extraction. The software reads the geometry of the face. Key factors include the distance between the eyes, the depth of eye sockets, the distance from forehead to chin, the shape of cheekbones, and the contour of the lips, ears, and chin. The aim is to identify the facial landmarks that are key to distinguishing the face.

3. The face capture process transforms analog information (a face) into a set of digital information (data) based on the person's facial features. Face's analysis is essentially turned into a mathematical formula. The numerical code is called a face print. In the same way that thumbprints are unique, each person has their own face print.

4. Feature matching: The features extracted from the original face image will be used in matching different classes of face images in the dataset in terms of a certain criterion, upon which the face classification into some class is performed.

Figure (1.4) shows the general procedure of a face recognition system. There are two main kinds of face recognition tasks [1, 15], face identification and face verification. In the identification, the input to the system is an image of an unknown person, and the system matches it to a gallery of known people. In the face verification, the input face image is an unknown with a claimed identity, and the system needs to decide whether the person is who he/she claims to accept or deny.



Graph (2.3) Face recognition system

CHAPTER THREE

THE PROPOSED SYSTEM

CHAPTER THREE
THE PROPOSED SYSTEM

3.1 Introduction

Face recognition has garnered considerable interest over the last three decades, and several research have been conducted in support of it. However, several obstacles remain.as the follow:

Frist method: We will examine the use of many approaches, including Principal Component Analysis (PCA) which is one of the most successful methods for modeling high-dimensional data in more and better ways and in fewer dimensions utilizing variables, therefore reducing the harm caused by information loss. Defined as the most straightforward technique of implementing those requirements in these dimensions. There are already available methods for representing the dispersed SRC can be align and correlate the facial pictures in the case of a capture error, and the dictionary matrix can be compared to the face images because each face image is treated as a vector in this dictionary matrix, just like the face images in the training set are. Each face image is represented as an image of a vector test face under this dictionary matrix. Which means that the sparse representation with a low rank is more representational and efficient.

Second method: We shall cross-reference (suggest) many effective techniques for face recognition, low-rank matrix (LR) and collaborative representation (CRC) were used to produce the aforementioned method. Some low-rank transformations in the images to interpret the context of face recognition. In this method, we can apply it directly to the original face image and it does not require feature selection and does not require many training samples.

Finally, we obtained results in several experiments, quantities and qualities, which show the work of those methods proposed above.

The remainder of this chapter is organized as follows: In Section 3.2, we review briefly some related works on low rank matrix recovery; in Section 3.3 sparse Coding; in Section 3.4 we give an overview of Robust Principal component analysis (RPCA); in Section 3.5 we briefly review the Sparse Representation based classification; in Section 3.6 we briefly review the Collaborative Representation Classifications (CRC).

3.2 Low-Rank Matrix Recovery

The term low-rank matrix refers to a method for constructing an uncertain matrix using a sample of its components with low-rank or near-low-rank constraints. However, in many situations, the matrix we are attempting to obtain is known to be constructed in a low-rank or near-low-rank fashion. The needs for inferring the global structure from a small number of local data drive this challenge. The nuclear norm may be used to minimize the rank function under generic circumstances (sum of singular values) [63]. Traditional interior point methods can only tackle problems with a few hundred variables since the natural reformulation of the nuclear norm results in a semi-definite program [64].

Recently an important sparse learning framework named robust principal component analysis (RPCA) [43] has been proposed. The nuclear norm has been minimized in matrix completion to solve a number of problems in computer vision and pattern recognition, including subspace alignment and RPCA.

3.3 Sparse Coding.

In recent years, a growing amount of study has been directed on finding things to learn via the use of sparse representation variables and their extensions. Wright et al. [43] developed a face recognition algorithm based on sparse representation-based classification (SRC). Each input test image is coded as a sparse linear mixture of these sample images using l_1 -norm reduction, and SRC utilizes training photos as a dictionary of primary indicators. Given training pictures for all k subjects, a dictionary matrix $D = (D_1, D_2 \ldots., D_K)$ as is constructed as where D_i is a subset of training samples for the subject and y is a sample image from the same topic. There are two stages to the original SRC algorithm:

(i) Sparsely code y on via l_1 -norm minimization

$\hat{\alpha}$= arg$min_\alpha \parallel y - D\alpha \parallel_2^2 + \lambda \parallel \alpha \parallel_1$ where λ is a scalar constant.

(ii) Do classification via$(y) = argmin_i\{\delta_i\}$ where δ_i= $\parallel y - D\widehat{\alpha_i} \parallel_2$

$\widehat{\alpha} = [\hat{\alpha}_1, \hat{\alpha}_2, \ldots, \hat{\alpha}_n]$and $\hat{\alpha}_i$ is the coefficient vector associated with subject i.

The robustness of SRC and its expansions is dependent on the method used, such as coding coefficients for dictionary learning. Whereas SRC imposes an unnatural organization, Collaborative Representation-based Classification (CRC) uses a normal organization, and Multiple Co-Sparse Representation-based Classification (MTJSRC) [65] uses a normal organization. The Fisher discriminant criteria was introduced using a combination of dictionary and coding discriminant parameters. To utilize discriminative information concealed in coding parameters, a dictionary learning technique named (Binary Discriminant Dictionary Learning) (BDDL) is suggested and natural organization is used.

3.4 Robust Principal Component Analysis (RPCA).

Principal component analysis is a widely used technique for data preprocessing, and it has undergone many investigations, modifications

, and adjustments over several decades, resulting in generalization or enhancement of certain of its unique features, one of which is robustness. According to the equation (2.1):

$$D = X + E$$

Where D ∈ $R^{m\times n}$ is a sparse matrix containing a specified large data matrix, X low-rank matrix, and E must be sparse.The easiest method is to use l_0-norm to minimize the energy-related function:

$$\underset{X,E}{\text{Min}}(rank\ (X) + \ \lambda \parallel E \parallel_0\ ), \quad \text{S.t} \quad D = X + E \tag{3.1}$$

Where λ is a freely chosen balancing parameter, yet this issue is NP-hard, a typical approach may entail a search of combinatorial complexity. To make this easier to solve, the obvious thing to do is to fix the minimization with l_0-norm, which generated an approximation convex problem:

$$\underset{X,E}{\text{Min}} \parallel X \parallel_* + \ \lambda \parallel E \parallel_i \quad , \text{S.t} \quad D = X + E \tag{3.2}$$

Where $\parallel X \parallel_*$ denotes the nuclear norm which is the singular value l_1-norm. Assuming these bare minimums, Given that the order of the low-rank and sparsity matrices is limited by the following equation, the (Principal Component Pursuit (PCP)) solution recovers fully the low-rank and sparse matrices:

$$rank(D) \leq \frac{P_r \max(n,m)}{(\mathcal{M}\ Log \min(n,m)^2} \qquad \parallel E \parallel_0 \leq P_E n, m \qquad (3.3)$$

Where P_r rand P_E are positive constants and m and n denote the dimensions of the matrix D. For the sake of explanation λ, is fixed $(1/\sqrt{\max(n,m)})$. The 0optimization method for Augmented Lagrange Multipliers (ALMs) [66] .was used to solve relation (3.3), because of their superior accuracy and speed when applied to real-world face pictures.

3.5 Sparse Representation Based Classification (SRC)

This section provides an overview of the Sparse Representation based Classification (SRC) method for face recognition [61]. Consider the issue of face identification using N training facial images drawn from. K classes, such as N=$\sum_{K=1}^{i} n_k$. $D = [D_1, D_2, \ldots, D_K] \in R^{m\times n}$ as adequate training examples for class i where D_i =[$v_{1,i}$,$v_{2,i}, \ldots, v_{in_i}$] $\in R^{m\times n}$. The testing face pictures $y \in R^m$ belonged to the same class as the training samples D_i, which can be expressed as:

$$y_i = \sum_{j=1}^{n_i} \alpha_{i,j} v_{i,j} \qquad (3.4)$$

For $i = 1, \ldots, m$, $j = 1, \ldots, n$

Because it is a dictionary, it contains all training samples D_i =[$v_{1,i}$,$v_{2,i}, \ldots, v_{i,n_i}$] and equation (3.4) can be rewritten into the for

$$y = D\alpha_0 \in R^m \qquad (3.5)$$

Where $\alpha_0 = \{0, \ldots, 0, \alpha_{i,1}, \alpha_{i,2}, \ldots, \alpha_{i,n_i}, \ldots, 0\}^T \in R^n$ is the coefficient vector in which all the majority of coefficients are zero save those corresponding to class$i$. With a high enough sample size for each class, the coefficient vector should be very sparse. The answer to may be retrieved by solving the following l_1-norm minimization problem [43], based on recent advancements in the ideas of compressed sensing and sparse representation.

$$\hat{\alpha}_1 = argmin \parallel \alpha \parallel_1 \quad \text{, s.t } D\alpha = y \tag{3.6}$$

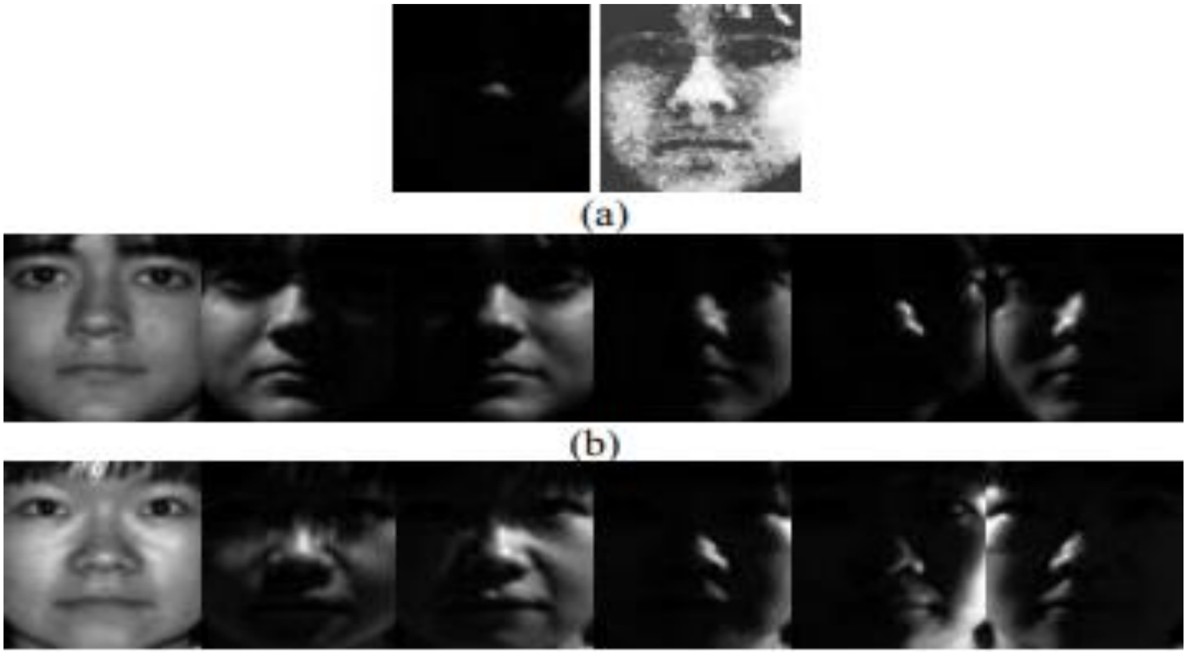
Where $\parallel \alpha \parallel_1$ denotes the l_1-norm of x which sums up the absolute approximate solution:

$$\hat{\alpha}_1 = argmin \parallel \alpha \parallel_1 \quad \text{s.t} \parallel D\alpha - y \parallel_2 < \varepsilon \tag{3.7}$$

This may be rephrased as the regularized values of all the items in α. Additionally, to account for the possibility that y contains dense noise, $D\alpha = y$ may be substituted by$\parallel D\alpha - y \parallel_2 < \varepsilon$, when there is a little margin of error. As a result, we may use the Lagrange multiplier to solve the corresponding convex relaxed optimization model:

$$\hat{\alpha}_1 = argmin\{\parallel D\alpha - y \parallel_2^2 + \lambda \parallel \alpha \parallel_2\} \tag{3.8}$$

Where$\lambda > 0$ the parameter is a scalar normalizing factor that finds a balance between sparsity and reconstruction error. As a consequence, for face recognition, the Sparse Representation-based Classification (SRC) [67], approach is as follows as shown in figure (3.1):



Figure(3.1): An example of class-specific face representation. (a) The query face image (left: original image; right: the one after histogram equalization for better visualization); (b) some training samples from the class of the query image; (c) some training samples from another class

---

Algorithm 3.1: Sparse Representation Based Classification

---

1. Input: Normalize the columns of X to have unit l_2-norm.
2. Code y over X via l_1-minimization

$$\hat{\alpha} = \arg min_{\alpha} \parallel \alpha \parallel_1 \quad \text{s.t} \parallel y - D\alpha \parallel_2 \ \leq \varepsilon$$

where constant ε is to $\varepsilon > 0$ account for the dense small noise in y, or to balance the coding error of y and the sparsity of α.

3. Compute the residuals

$$r = \| y - D\widehat{\alpha} \ \|_2.$$

where $\widehat{\alpha}_i$the coding coefficient vector is associated with class i.

4. Output the identity of y as Identity (y) =$argmin_i \{r_i\}$, for $i = 1, \ldots, m$.

3.6 Combination of RPCA and Sparse Representation-Based Face Recognition:

This section describes a technique for identifying faces in the presence of lighting variations, corruption, and pixel blockage. We have adopted RPCA to conduct face recognition. To be more precise, we align a test facial image with the training photographs for each subject first, then analysis of the main components of strong works on images of each topic separately to provide low-rank images that match the different image test subjects. The low-rank component includes critical discrimination information, which is required to counteract identification.

Algorithm 3.2: LR-SRC

1. Input: A matrix of normalized training samples $X = [X_1, X_2, \dots, X_i] \in R^{m\times n}$ at testing sample $Y = [Y_1, Y_2, \dots, Y_i\ ] \in R^{m\times n}$ for k classes.
2. for each subject i do
3. Using algorithm (ALM)[66] to perform RPCA on X_i, Y_i

   $min_{A,S}\|A_i\|_* + \lambda\|S_i\|_1 \quad s.t \quad X_i = A_i + S_i$

   $min_{A,S}\|B_i\|_* + \lambda\|E_i\|_1 \quad s.t \quad Y_i = B_i + E_i$
4. Low rank matrices $A = A_i$ and $B = B_i$, sparse matrices $S = S_i$ and $E = E_i$
5. Normalize the columns of A to have unit $l_2 - norm$.
6. Code B over A by $\hat{\alpha} = \arg min_\alpha \|\alpha\|_1$ s.t $\|B - A\alpha\|_2 < \varepsilon$.
7. Compute the residuals

   $e_i(B) = \|B - A \cdot \hat{\alpha}_i\|_2$

   Where $\hat{\alpha}_i$ is the coding coefficient vector associated with class i.
8. Output the identity of B as identity(B) $=$ arg$min_i(e_i)$.

3.7 Collaborative Representation Classifications (CRC)

We assume n unique bases D = $[D_1, D_2, \dots, D_n] \in R^{m\times n}$, which are drawn from N distinct classes, where m denotes the dimension of each base. Class i contains n_i training pictures indicated by the symbolD_i. D can be rewritten as D = $[D_1, D_2, \dots, D_n]$. When a fresh test sample y $\in R^m$ is encountered, SRC attempts to identify a sparse linear representation coefficient vector α ∈ R^n such that y = Dα may be represented. Calculate this approximation problem by minimizing the following issue relation (3.8):

$$\hat{\alpha}= \text{arg}min_{\alpha}\{\| \ y - \text{D}\alpha \ \|_2^2 + \ \lambda \ \| \ \alpha \ \|_1\}$$

where λ is a scalar constant, $\|\cdot\|_2$ is the l_2 norm, and $\|\cdot\|_1$ is the l_1 norm. Numerous methods, such as basis pursuit[63] and Homotopy[64], may be utilized to solve the $l_1$norm minimization issue described above. The test sample y should be contained inside the space bounded by the proper class's training samples.

Once we have the solutions relation $\hat{\alpha}$ (3.8), where $\hat{\alpha} = [\alpha_{1,}\alpha_2, \dots, \alpha_N]$ and α_i is the representation vector $\hat{\alpha}$ for class i, the test sample y may be identified by the reconstruction error of each class, i.e.

$$\text{identity(y)} = \text{arg } min_i\{\|\text{y} - X_i\widehat{\alpha_\iota}\ \|_2\} \qquad (3.9)$$

Because various individuals' facial image may look similar, samples from uncorrelated classes may contribute to portraying the test sample y. CRC is a regularized least squares technique with a substantially reduced complexity that is defined as,

$$\hat{\rho} = \text{Arg}\, min_{\rho}\{\| y - \text{X}\rho \|_2^2 + \lambda \| \rho \|_2^2\} \quad (3.10)$$

Where λ is a balance factor. CRC can offer improvements in decreasing computational complexity by using the l_2-norm-based model. A closed-form solution:

$$\hat{\rho} = (X^T \text{X} + \lambda \text{I})^{-1} X^T y \quad (3.11)$$

May be obtained by solving relation (4.8), in which $(X^T \text{X}+\lambda\text{I})^{-1}X^T$ can be pre-calculated, resulting in rapid CRC computation speed. The regularized residuals

$e_i = \| y - X_i\widehat{\rho_i} \|_2 / \| \widehat{\rho_i} \|_2$" are utilized in the classification stage to categorize the test image y using the discriminating information contained in $\| \widehat{\rho_i} \|_2$, where $\widehat{\rho_i}$ is the coefficient vector associated with class i.

Finally, the test sample belongs to the class with the smallest regularized residual.

Algorithm 3.3: Collaborative Rrepresentation Based Classification [67]

1. Normalize the columns of X to have unit l_2-norm.
2. Code Y over X by

$$\hat{p}=py$$

Where $p = (X^T \text{ X}+\lambda\text{I})^{-1}X^T y$

3. Compute the regularized residuals

$$e_i = \| y - X_i\widehat{\rho_i} \|_2 / \| \widehat{\rho_i} \|_2$$

4. Output the identity of y

$$\text{identity(y) = arg } min_i(e_i)$$

3.8 Contribution Low Rank Matrix and CRC for Face Recognition

The suggested implementation of the method here in the presence of the creation of a new algorithm for facial recognition in two stages is presented in this section:

First, in the presence of light fluctuations, pixel corruption, and continuous occlusion, low-rank matrix recovery was used. Set a test face image, in particular, for each subject we first put it with the training. Under various subjects, the appropriate low-rank matrix recovery has been obtained with regard to the test image for each subject's photos separately.

Second, for low-rank image, collaborative representation-based classification (CRC) is implemented. The process may be summarized as follows: algorithm 4.2.

Algorithm 3.4: LR-CRC

1. Input:A matrix of normalized training samples$X = [X_1, X_2, \dots, X_i\ ] \in R^{m \times n}$ at testing sample $Y = [Y_1, Y_2, \dots, Y_i\ ] \in R^{m \times n}$ for k classes.
2. For each subject i do
3. Using algorithm (ALM)[66] to perform RPCA on X_i, Y_i

   $min_{A,S} \|A_i\|_* + \lambda \|S_i\|_1 \quad s.t \quad X_i = A_i + S_i$

   $min_{A,S} \|B_i\|_* + \lambda \|E_i\|_1 \quad s.t \quad Y_i = B_i + E_i$
4. Low rank matrices $A = A_i$ and $B = B_i$, sparse matrices $S = S_i$ and $E = E_i$
5. Normalize the columns of A to have unit $l_2 - norm$.
6. Code B over A by $\hat{\rho} = PB$ where $P = (A^T A + \lambda \cdot I)^{-1} A^T$.
7. Compute the regularized residuals

   $e_i = \|B - A \cdot \hat{\rho}_i\|_2 / \|\hat{\rho}_i\|_2$.
8. Output the identity of B as $\text{identity(B)} = \text{arg}min_i(e_i)$.

3.9 Discussion and Analysis

In this chapter, we have introduced the RPCA and (LR-SRC, LR-CRC), and some application of this model in image analysis, focusing on face recognition application. The applications of this is that sparse representation-based classification can be able to deal with the illumination variation and expression change and has discriminative ability when handling occlusion. Such knowledge could be used for many purposes like SRC paradigm assumes that there are sufficient training samples in each class so that the test image could be well represented with samples in a class where it belongs in fact, this is not always the case in the face recognition system. As well as the nature of corrupted and occluded regions. we be used purposes LR-CRC paradigm assumes .The method can be applied directly to original face image neither does it require feature selection, nor does it need many training samples.

CHAPTER FOUR

THE EXPERIMENTAL TEST AND RESULTS

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THE EXPERIMENTAL TEST AND RESULTS

4.1 Introduction

The performance of the proposed method in this thesis is evaluated by conducting a set of experiments done on the Face Recognition Dataset. These experiments and their results are described in full detail in this chapter. The experiment is conducted using a computer running Windows10 operating system with an Intel® Core™ i7-7700HQ processor with a memory of 16GB. The method is implemented and evaluated using the Matlab programming language.

4.2 Face Recognition Database

We have performed three different types of experiments to verify capabilities of the proposed approach. In the first one, Face Recognition task is executed. In the second one, Gender Recognition is implemented. Finally, the last experiment is carried out over a Facial expression. For all experiments there are three parameters in multi-block RCR: and (the Lagrange multiplier of the entropy constraint), it should be set those experiments on four of the most popular face database benchmarks: (1) The Olivetti Research Laboratory (ORL) face database, (2) AR face database, (3) Extended Yale B face database, and (4) UMIST face database. The following sections go over the specifics of the experiments and their outcomes.

4.3 Experimentation results

The Olivetti Research Laboratory (ORL) face database consists of 400 image database of faces. There are 10 different images of each of 40 distinct subjects, and for subjects, the images were taken by varying the lighting, facial expressions (open / closed eyes, smiling / not smiling) and facial details (glasses / no glasses). In our experiment, the first 4 images from each person were used for training and the other 6 images are used for testing.

Figure (4.1) Sample images from ORL face database

AR face database comprises over 4,000 images corresponding to 100 individuals. For each individual. Images featured frontal view faces, together with different facial expressions, illumination conditions, and occlusions (sunglasses and scarf). Our experiment is conducted with a subset composed of 50 male subjects and 50 female subjects. For each subject

Figure (4.2) Sample images from AR face database

The Extended Yale B face database [68] is composed of 2414 frontal facial images of 38 people, about 64 images for each subject. The images were taken under laboratory-controlled variable illumination conditions. 15, 20, 25 images for every person, randomly selected, are used for training, while the remaining images are used for testing.

Figure (4.3) Sample images from Extended Yale B face database

The UMIST face database contains 575 images of 20 distinct subjects (mixed race/gender/appearance). Each individual is shown in a range of poses from profile to frontal views. It is challenging in computer vision because the variations between the images of the same face in viewing direction are almost always larger than image variations in face identity. The data was split up with 15 samples per person were randomly selected for training and the rest for testing.

Figure (4.4) Sample images from UMIST face database

4.4 Results Evaluation

The results of our experiments on the ORL face database are presented in Table 4.1. The table shows the comparison of our proposed method with low rank Sparse Representation based Classification (LR-SRC) method, Sparse Representation based Classification (SRC) and Collaborative Representation based Classification (CRC) methods. We note that our proposed method achieves recognition rate better than the maximal recognition rates that have been achieved by the other methods in the same feature dimensions.

Table 4.1 Recognition rate (%) of different methods on the ORL database and the associated dimension of feature.

Method	SRC	CRC	LCR	RCR	**LR-SRC**	**LR-CRC**
Rate	90.0	89.17	85.60	82.08	**95.02**	**94.60**

The results of the experiments on AR face database are presented in Table 4.2. we are comparison of our proposed method low rank for Sparse Representation based

Classification(LR-SRC) and Nearest Neighbor (NN) Classifier using cosine distance methods were compared [70]. The results presented in Table 4.2 show that our proposed method leads to better recognition rate than highest recognition rate was obtained by the other methods with the same dimensions of features.

Table 4.2 Recognition rate (%) of different methods on the AR database and the associated dimension of feature.

Method	SVM	SRC	CRC	LRC	MDTJSR	RCR	**LR-SRC**	**LR-CRC**
Rate	87.1	93.7	93.3	76.4	95.8	95.9	**93.28**	**96.13**

Table 4.3, we present the results of our experiments on the Extended Yale B face database. By comparison, our proposed method with Low Rank Sparse Representation based Classification (LR-SRC) method; (SRC) and CRC Classifier and other methods, the results of the experiments on the Extended Yale B face database suggest that the recognition rate in our proposed method is better than the highest recognition rate presented by the other methods in the same feature dimensions.

Table 4.3 Recognition rate (%) of different methods on the Extended Yale B database and the associated dimension of feature.

method	15	20	25
SVM	67.1	76.5	87.1
SRC	84.6	91.3	92.0
CRC	84.7	91.3	92.4
LRC	81.8	87.0	89.0
MDTJSR	87.2	91.5	93.6
RCR	87.2	93.3	93.6
LR-SRC	**92.14**	**94.18**	**96.93**
LR-CRC	**93.27**	**95.55**	**98.02**

The experimental results on the UMIST face database are shown in Table 4.4, which demonstrates comparison of our proposed method with Extreme Learning Machine (ELM) [69], Sparse Representation based Classification (SRC) [67], Extreme Learning Machine with Sparse Representation based Classification (ELMSRC) methods were compared in [70] and Collaborative Representation based Classification (CRC) methods. It can be inferred from that the performance of our proposed method is better than performance of the other methods.

Table 4.4 Recognition rate (%) of different methods on the UMIST database and the associated dimension of feature

Method	SRC	CRC	ELM	ELM SRC	**LR-SRC**	**LR-CRC**
Rate	98.33	98.18	96.51	98.36	**99.64**	**99.27**

Chapter Five

Conclusions and Future Works

CHAPTER FIVE

Conclusion and Further Work

5.1 Conclusions

In this thesis,

1. Several low-rank sparse representation methods have been developed and the dictionary matrix has been enhanced developed improved algorithms, and the benefit of sparse representation is to study the basic problem of face recognition, and in several different conditions for facial imaging.
2. The basic models of classification based on scattered representation is a linear mixture through some training samples and has great potential to build representation, including samples with the undefined test.
3. We made a sophisticated improvement to the accuracy of the performance of these methods by using face images, and some spare representation with low ranks are represented, and high quality of key components (RPCA).
4. The original image samples that have a lot of repetitions and add to that noise and some trivial information are unrecognizable and this is not a positive thing in that place.
5. If the training samples are in large numbers, the calculation of the SRC takes a long time, and to address this we used RPCA as processing steps in the past.

Finally, we improved an algorithm for face recognition, where the collaborative representation based low rank matrix recovery which is an efficient technique, has been proposed for image feature extraction and classification. Since the proposed approach is based on the image matrix, it is easier to use for image feature extraction.

In this approach, a low rank matrix of the image is formulated using robust principal component analysis (RPCA) to access the actual representation features of image. The procedure of Collaborative Representation model (CRC) is applied to low rank matrix, which effectively exploits the similarity and distinctiveness of different features for coding and classification. The feasibility of the proposed method is demonstrated on different face databases. It gives better performance, such as recognition performance in comparison with the reported face recognition methods.

5.2 Future Works

The algorithms that we worked on were profitable results of high quality, but they took many directions to improve those algorithms that specialize in the framework of scattered representation. Those proposed algorithms can be tested in our work on several databases that are difficult to obtain due to lack of time and some current conditions.

The shape and design of the proposed algorithm is simple, which makes it attractive and tempting to other biometric applications, and based on the results provided by the proposed algorithm, three-dimensional biometrics can be treated, and this is done by developing a new concept for classifying the ears of three-dimensional faces, and so on. There are many practical problems, including image classification and replay. Build its image and identify the bio-textile by adapting the proposed algorithms.

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المستخلص

يشير التعرف على الوجوه عمومًا إلى فئة من الأساليب التي تحل المشكلات من خلال تمثيل المتغيرات المهمة كمصفوفات ذات رتبة منخفضة. لقد حقق نجاحًا كبيرًا في مختلف المجالات بما في ذلك رؤية الكمبيوتر / واختلافات الاضاءة / واختلافات التعبير الشديدة لوجه / وفساد البكسل العشوائي / ظلال وتمويه والمعلوماتية الحيوية.

في الآونة الأخيرة / تم إحراز تقدم كبير في النظريات والخوارزميات والتطبيقات ذات الرتبة المنخفضة للتعرف على الوجوه ، مثل مصفوفة الرتبة المنخفضة للتعرف على الوجوه المطبقة على التمثيل المتناثر والتمثيل التعاوني. جلبت هذه التطورات المزيد والمزيد من الاهتمام بهذا الموضوع. في هذه الأطروحة نقترح أن نراجع التقدم الأخير في الرتبة المنخفضة للتعرف على الوجوه / وأحدث الخوارزميات / والتطبيقات ذات الصلة في تحليل الصور. نقدم أولاً نظرة عامة على مفهوم الرتبة المنخفضة للتعرف على الوجوه والمشكلات الصعبة في هذا المجال. بعد ذلك ، نلخص النماذج والخوارزميات لمصفوفة الرتبة المنخفضة في التعرف على الوجوه ونوضح مزاياها وقيودها من خلال التجارب العددية. بعد ذلك ، نقدم بعض التطبيقات ذات الرتبة المنخفضة للتعرف على الوجوه في سياق تحليل الصور

وزارة التعليم العالي والبحث العلمي

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وهي جزء من متطلبات نيل درجة الماجستير في علوم الرياضيات

من قبل

رسل اسماعيل خليل

المشرف

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